Langmuir-probe characteristic in the presence of drifting electrons

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The electron current to a negatively biased, cylindrical Langmuir probe is computed for the case of drifting, Maxwellian electrons in a low-pressure discharge. The degree of drift is characterized by $v_d/v_{\rm th}$, where v_d and $v_{\rm th}$ are the electron drift and thermal speeds, respectively. We find that the inflection point of the characteristic underestimates the plasma potential, for $v_d/v_{\rm th}>1.25$. Further, for $v_d/v_{\rm th}$ greater than about 0.1, an analysis assuming stationary electrons computes an erroneously high temperature. Probe data from a sputtering magnetron discharge are fitted with the drifting model, giving a concrete example of its use.

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I. INTRODUCTION

Langmuir-probe diagnostics [1,2] provide much useful information about many of the plasmas important in both basic plasma research and plasma processing. Typically, determining plasma parameters from the probe characteristic (i.e., the current collected from the plasma by the probe vs the probe bias) is an inverse problem. Functional forms for the electron and ion distribution functions are assumed, and the current collected from these distributions is calculated as a function of the model parameters [3] (e.g., for a stationary distribution function these parameters are the density and temperature). Plasma parameters are then inferred by fitting the model current to the probe characteristic. Unfortunately, if the wrong model is used, the calculated plasma parameters will be errant. Most probe theory considers the case of a plasma at rest [4]. However, in some plasmas the electron component may be drifting with respect to the probe (i.e., the first moment of the electron distribution function is nonzero). Under this circumstance it is important to recognize the signature of these drifting electrons in the probe characteristic so that the characteristic may be correctly analyzed.

Expressions predicting the current collected by a probe immersed in a plasma where some components have nonzero drift velocities have long been known. In particular, models have been developed for both high- [5,6] and low-pressure [3,7] cases. Here we consider the low-pressure case (i.e., collisions in the sheath are negligible) first discussed by Mott-Smith and Langmuir [3] and later considered more thoroughly for the ion component by Heatley [7]. We restrict our treatment to that of the electron current collected by a probe biased below the plasma potential (i.e., the retarding field region). These results are useful for the interpretation of probe characteristics where the probe is stationary and the electrons are drifting, or where the probe is moving and the electrons are stationary, as in space applications.

In Sec. II, expressions for the electron current collected by a planar probe are reviewed. In Sec. III, we extend our discussion to the cylindrical case and consider the errors made if probe characteristics from a plasma with drifting electrons are interpreted using the stationary model. These errors include overestimating the electron temperature and underestimating the plasma potential. In Sec. IV, we provide a concrete example of the application of this model by fitting data from a sputtering magnetron discharge. Agreement with the drifting model is found to be quite good.

II. PLANAR PROBE

In this section we consider the electron current collected by a planar probe in a plasma with drifting electrons. These results are extended in the next section to the important case of a cylindrical probe.

Consider the current collected by a one-sided, perfectly absorbing, planar probe. The z axis is perpendicular to the probe face and points to it. That is, electrons with a z component of velocity $v_z>0$ are moving toward the probe face, and those with $v_z<0$ are moving away. The probe is immersed in a plasma where the electron distribution function is given by $f(v_x,v_y,v_z)$. The integral of f gives the electron density n. The probe is biased below the plasma potential to retard electrons.

The current density of electrons at the probe face as a function of the retarding bias can be found as follows. Only those electrons with $\frac{1}{2}mv_z^2\geq -eV_r$ will be collected by the probe. Here m is the electron mass, $-e$ is the electron charge, and V_r is the retarding probe bias taken with respect to the plasma potential V_p. Thus the electron current density at the probe face is given by

$$J=-e\int_{-\infty}^{\infty}dv_x\int_{-\infty}^{\infty}dv_y\int_{\sqrt{-2eV_r/m}}^{\infty}dv_z v_z f(v_x,v_y,v_z)\ . \tag{1}$$

To calculate J, the electron distribution function must be specified. We assume a drifting Maxwellian distribution with a single temperature T,

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$$f(v_x,v_y,v_z)=\frac{n}{\pi^{3/2}v_{\rm th}^2}\exp\left[-\frac{(v_x-v_{dx})^2+(v_y-v_{dy})^2+(v_z-v_d)^2}{v_{\rm th}^2}\right], \tag{2}$$

where $v_{\rm th}=\sqrt{(2T/m)}$ is the thermal velocity for T having units of energy, and v_{dx}, v_{dy}, and v_d are the x, y, and z components of the drift velocity, respectively. The electron current collected by the probe is computed by substituting the electron distribution function [Eq. (2)] in the current density integral [Eq. (1)]. The integral yields

$$J=-en\left\{\frac{v_{\rm th}}{2\sqrt{\pi}}\exp\left[-\left[\frac{\sqrt{-2eV_r/m}-v_d}{v_{\rm th}}\right]^2\right]+\frac{v_d}{2}\operatorname{erfc}\left[\frac{\sqrt{-2eV_r/m}-v_d}{v_{\rm th}}\right]\right\}, \tag{3}$$

where erfc(x) is the complementary error function [8] [i.e., erfc$(x)=1-$erf(x)]. Thus the current density has both exponential and error function dependencies on the retarding bias. Note that J depends only on the component of the drift velocity perpendicular to the probe face v_d. Consequently, the probe registers a stationary distribution if the direction of the electron drift velocity is parallel to the probe face.

When $v_d=0$, the collected current density has the well-known exponential dependence on probe bias,

$$J=-en\frac{v_{\rm th}}{2\sqrt{\pi}}\exp\left[\frac{eV_r}{T}\right], \tag{4}$$

and the current density collected when the probe is biased at the plasma potential (i.e., $V_r=0$) is

$$J_0=-en\frac{v_{\rm th}}{2\sqrt{\pi}}. \tag{5}$$

The collected current density [Eq. (3)] is conveniently normalized by J_0 to give

$$\frac{J}{J_0}=\exp\left\{-\left[\left[\frac{-eV_r}{T}\right]^{1/2}-\frac{v_d}{v_{\rm th}}\right]^2\right\}+\sqrt{\pi}\frac{v_d}{v_{\rm th}}\operatorname{erfc}\left[\left[\frac{-eV_r}{T}\right]^{1/2}-\frac{v_d}{v_{\rm th}}\right]. \tag{6}$$

This normalized current density [Eq. (6)] depends on two dimensionless variables. The electron drift velocity is normalized by the electron thermal velocity, so that $v_d/v_{\rm th}$ is the dimensionless drift velocity. The probe bias is normalized by the electron temperature, giving eV_r/T as the dimensionless retarding bias.

III. CYLINDRICAL PROBE

A. Collected current

Consider a cylindrical probe, as shown in Fig. 1. End effects are assumed to be negligible. The electrons in the plasma have a nonzero drift velocity with respect to the probe. However, the probe registers only the component of the drift velocity perpendicular to its axis. We call this perpendicular component of the drift velocity v_d. As we assume that T is isotropic, different angles θ around the probe's surface correspond to different projected values of the drift velocity. Consequently, the normalized current density J/J_0 can be written as an average [9] of the planar current density [Eq. (6)] [7]:

$$\frac{J}{J_0}=\frac{1}{2\pi}\int_0^{2\pi}d\theta\left[\exp\left\{-\left[\left[\frac{-eV_r}{T}\right]^{1/2}-\frac{v_d}{v_{\rm th}}\cos\theta\right]^2\right\}+\sqrt{\pi}\frac{v_d}{v_{\rm th}}\cos\theta\operatorname{erfc}\left[\left[\frac{-eV_r}{T}\right]^{1/2}-\frac{v_d}{v_{\rm th}}\cos\theta\right]\right]. \tag{7}$$

Note that, as in the planar case, J/J_0 depends on two dimensionless parameters: the dimensionless drift velocity $v_d/v_{\rm th}$ and the dimensionless retarding bias eV_r/T. If there is no net electron drift, or if the drift velocity is parallel to the probe axis so that $v_d=0$, then Eq. (7) reduces to the stationary expression of Eq. (4). Though no closed form solution of Eq. (7) is known, a solution in the form of an infinite series of Bessel functions has been reported [7]. Alternatively, Eq. (7) can be evaluated via direct numerical integration, and this is our approach.

In Fig. 2 we plot the normalized current density J/J_0 against the dimensionless retarding bias for various values of the dimensionless drift velocity. When $v_d/v_{\rm th}\ll 1$, the current increases exponentially as expected. From Fig. 2, we see that for $v_d/v_{\rm th}\lesssim 10^{-1/2}=0.32$, the curves are practically indistinguishable. As $v_d/v_{\rm th}$ becomes larger, the increase in the collected electron current for increasing retarding potentials is weaker than the exponential. Consequently, the curvature of the characteristic decreases with increasing $v_d/v_{\rm th}$. This reduced curvature in the characteristic is one indication that the plasma contains a drifting electron distribution.

Furthermore, as seen in Fig. 2, the amount of electron current collected at a given probe bias increases with $v_d/v_{\rm th}$. As we have defined v_d as the component of the electron drift velocity perpendicular to the probe axis, the collected electron current will be smallest when the axis of the probe is parallel to the direction of the electron drift velocity and greatest when the probe axis is perpendicular to the direction of drift. Consequently, we expect that the floating potential of the probe will be most negative [10] when the probe axis is perpendicular to the electron drift velocity and least negative when it is parallel. This observation may give a simple method of determining the direction of the electron drift velocity.

B. Plasma potential

The plasma potential V_p is given by the inflection point of an ideal probe characteristic. That is, the plasma potential is given by the probe bias where dJ_p/dV_r is largest and where J_p is the total current density drawn by the probe. In real life, surface contamination [11] may greatly reduce the collection of low-energy electrons, so that the inflection point falls below the true plasma potential. (We assume that the contribution of the electron current to the derivative is much greater than that of the ions due to the mass ratio.) For a cylindrical probe immersed in a plasma with a drifting electron distribution, the normalized derivative of the electron current in the retarding field region is

$$\frac{d(J/J_0)}{d(eV_r/T)}=\frac{1}{2\pi}\int_0^{2\pi}d\theta \exp\left\{-\left[\left[\frac{-eV_r}{T}\right]^{1/2}-\frac{v_d}{v_{\text{th}}}\cos\theta\right]^2\right\} . \tag{8}$$

This derivative is plotted against the normalized probe bias in Fig. 3. For $v_d/v_{\text{th}}=0$, the derivative has the expected exponential dependence. Further, all curves except $v_d/v_{\text{th}}=10^{1/4}=1.78$ increase monotonically. For the $v_d/v_{\text{th}}=10^{1/4}$ curve, an inflection point in the electron current is found below the plasma potential. For the other curves, there is no inflection point for $V_r<0$. Consequently, as Heatley [7] recognized, the inflection point of the probe characteristic in the presence of drifting electrons may underestimate the plasma potential. Also note that the peak in the derivative for $v_d/v_{\text{th}}=10^{1/4}$ is broad and symmetric about its maximum. This broadening in the derivative is another signature of drifting electrons.

In Fig. 4 we plot the location of the inflection point in the electron characteristic as a function of the normalized drift velocity. When there is no inflection point in the retarding field region, we take the inflection point as the plasma potential, $eV_r/T=0$. For $v_d/v_{\text{th}}>1.25$, the inflection point moves below the plasma potential, as seen by the break in the plotted curve at $v_d/v_{\text{th}}=1.25$. Consequently, if $v_d/v_{\text{th}}>1.25$, the inflection point of the characteristic underestimates the plasma potential. However, when the drift velocity is known the shift in the position of the inflection point can be found using Eq. (8), and the correct plasma potential can be computed.

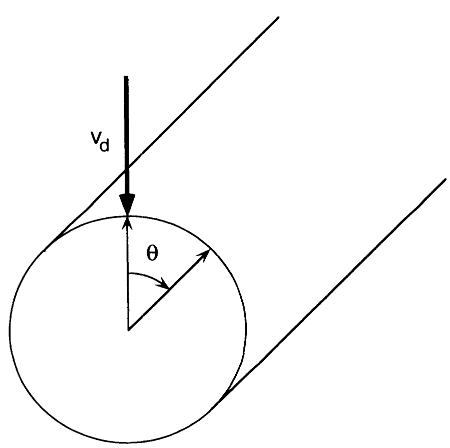

FIG. 1. Coordinate system for the cylindrical probe. The probe is immersed in a plasma with a drifting electron distribution. The maximum electron drift velocity projected perpendicular to the probe axis is v_d. The current density at the probe's surface is found by integrating over θ [Eq. (7)].

C. Electron temperature

For a stationary Maxwellian distribution, the electron temperature is given by the inverse of the derivative of the logarithm of the collected electron current evaluated at $V_r=0$ (i.e., at the plasma potential). That is,

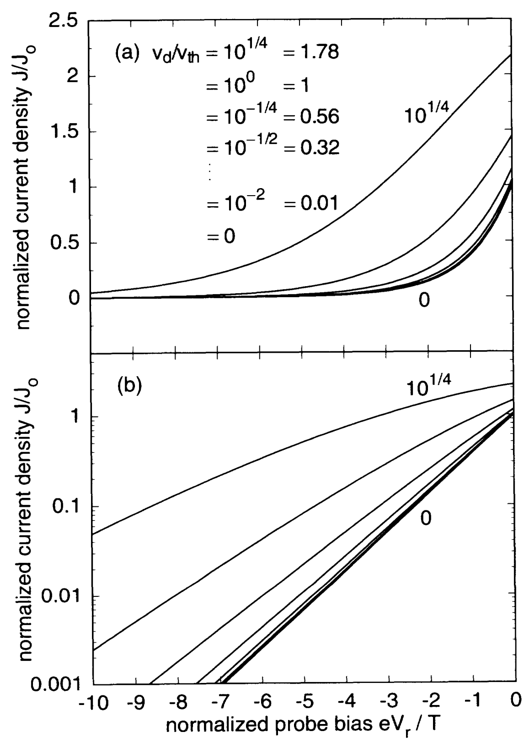

FIG. 2. Plots of the normalized electron current density J/J_0 [Eq. (7)] for a cylindrical probe vs the normalized retarding potential eV_r/T for various values of the normalized drift velocity v_d/v_{th}. Curves are plotted on linear axes in (a) and on semilogarithmic axes in (b). The top curve is for $v_d/v_{\text{th}}=10^{1/4}$, and the curves continue downward in a progression of powers of $10^{1/4}$. Note that as v_d/v_{th} increases, the collected current increases and the curvature of the characteristic decreases.

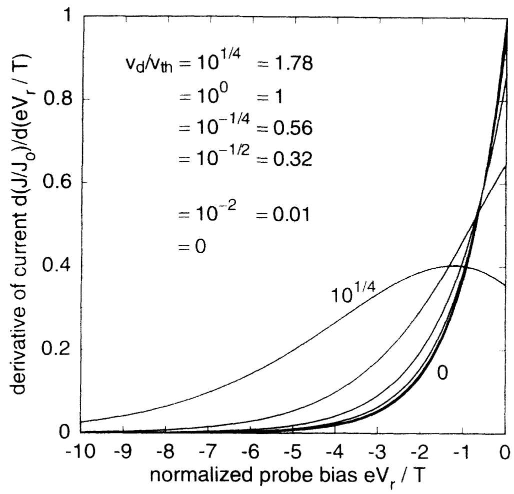

FIG. 3. Derivative of the electron current density collected by a cylindrical probe [Eq. (8)] vs the normalized retarding bias eV_r/T for various values of the normalized drift velocity v_d/v_{th}. Note that the maximum of the derivative is at $eV_r/T=0$ (i.e., the plasma potential) for $v_d/v_{th}=0$, but moves to a lower value for $v_d/v_{th}=10^{1/4}$. Consequently, the inflection point of the characteristic will underestimate the actual plasma potential above some threshold value of v_d/v_{th}.

$$\frac{e}{T_{\text{slope}}}=\frac{d(\ln J)}{dV_r}\bigg|_{V_r=0}, \tag{9}$$

where T_{slope} is the temperature estimated in this way. Though this is not a recommended way of estimating T from a probe characteristic, it allows a convenient comparison between the drifting and stationary models, and gives an upper limit on the error introduced by applying the stationary model to a probe characteristic taken in a plasma with drifting electrons.

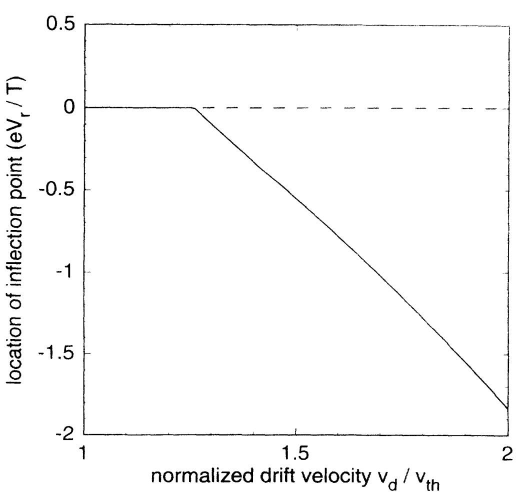

FIG. 4. Position of the inflection point in the electron characteristic vs probe bias as a function of the normalized drift velocity v_d/v_{th}. For $v_d/v_{th}<1.25$, there is no inflection point in the characteristic in the retarding field region. However, for $v_d/v_{th}>1.25$, the inflection point moves below the plasma potential.

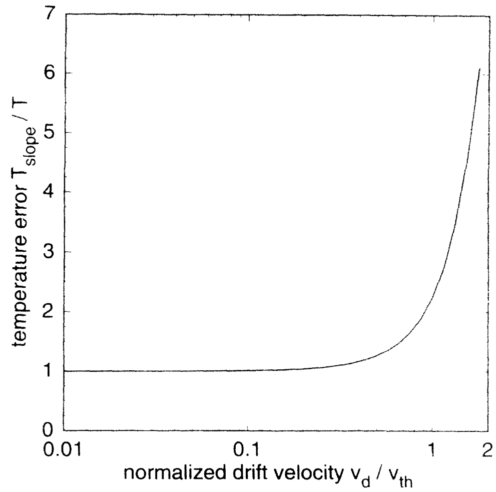

FIG. 5. The ratio of the electron temperature predicted by the stationary model T_{slope}, to the actual electron temperature T [Eq. (10)] plotted against the logarithm of the dimensionless drift velocity v_d/v_{th}. As v_d/v_{th} increases, the discrepancy between the correct temperature and the temperature predicted by the stationary model grows rapidly. Consequently, for an electron drift velocity comparable to the thermal velocity, the temperature found using the stationary model significantly overestimates the true electron temperature.

For a drifting distribution, Eq. (9) overestimates the electron temperature. The ratio of T_{slope} to the correct temperature T can be written in terms of the normalized current density and its derivative as

$$\frac{T_{\text{slope}}}{T}=\left[\frac{1}{J/J_0}\frac{d(J/J_0)}{d(eV_r/T)}\bigg|_{V_r=0}\right]^{-1}. \tag{10}$$

In Fig. 5 we display the dependence of T_{slope}/T on v_d/v_{th}. For $v_d/v_{th}\ll 1$, $T_{\text{slope}}/T\approx 1$, and the temperature is accurately predicted. For $v_d/v_{th}=0.10$, T_{slope} overestimates T by 1%, and, for $v_d/v_{th}=0.32$, T_{slope} is 10% greater than T. The discrepancy grows rapidly for $v_d/v_{th}>0.1$, and, for $v_d/v_{th}=1.0$, T_{slope} is 2.2 times greater than the actual temperature. Consequently, using the stationary model, a non-negligible electron drift is interpreted as an erroneously high temperature.

IV. COMPARISON WITH EXPERIMENT

The cylindrical probe theory for drifting electrons presented in this paper has been used to fit data taken in a cylindrically symmetric, planar, sputtering magnetron discharge [12]. Because of the crossed electric and magnetic fields [13] in this device, electrons undergo a strong $\mathbf{E}\times\mathbf{B}$ drift. (Ions are unmagnetized since the ion Larmor radius is much larger than the characteristic magnetic field size.)

Data were taken at a neutral argon pressure of 0.42 Pa, using a copper cathode. The discharge voltage was -400 V dc and the discharge current was 64 mA. To characterize the discharge, a cylindrical tungsten probe with a diameter of 0.254 mm and a length of 3.0 mm was used. The probe was located 1.9 cm from the symmetry axis and 1.3 cm above the cathode, with the axis of the probe on a radial chord. Consequently, the axis of the probe was perpendicular to the $\mathbf{E}\times\mathbf{B}$ drift direction. At the probe tip, $B\approx 70$ G.

For comparison, data in the retarding field region of the probe characteristic were fit with models appropriate for either drifting or stationary electron distributions [Eqs. (4) and (7), respectively]. The ion current was modeled by a term proportional to [14] $\sqrt{V_r}$ (i.e., cold orbit-limited ion motion was assumed). As both models are nonlinear, a multidimensional downhill Simplex method [15] was used to minimize the sum of the squares of the errors, subject to the constraints that n, T, $v_d \geq 0$.

The fit to the drifting electron model is shown in Fig. 6, where we compare the total probe current to our model, including the ion current term. Initially, we found $V_p=2.35$ V, $T=1.59$ eV, $n=7.02\times10^{15}$ m^{-3}, and $v_d=1.10\times10^6$ m/s. However, since for these parameters $v_d/v_{th}=1.44>1.25$, the plasma potential must be corrected and the data fitted again with the new estimate for V_p. After 54 iterations, plasma parameters converged to $V_p=-0.038$ V, $T=1.33$ eV, $n=8.22\times10^{15}$ m^{-3}, and $v_d=1.34\times10^6$ m/s , so that $v_d/v_{th}=1.93$. Note that V_p, n, and v_d have increased with respect to the initial values, whereas T has decreased. The plasma potential found in this way is about 2.3 V greater than that estimated from the inflection point of the probe characteristic. The presence of an azimuthal electron drift was confirmed using a single-sided planar probe, though because of it larger size it was not possible to make measurements near the cathode. Using $B=$ 70 G and $v_d=1.34\times10^6$ m/s, we find $E\approx 90$ V/cm, assuming we are measuring the $\mathbf{E}\times\mathbf{B}$ drift velocity.

For comparison, the fit to the stationary model is shown in Fig. 7. Note that the agreement is poor. The stationary fit repeatedly crosses the data to minimize the error. For this fit we assumed that the plasma potential was given by the inflection point $V_p=-2.35$ V. The electron temperature and density found were $T=4.47$ eV and $n=8.30\times10^{15}$ m^{-3}, respectively. Agreement of these parameters with the ones found using the drifting model is poor. In particular, the temperature is 3.3 times larger than that found with the drifting model. As noted in Sec. III C, this discrepancy is less than we would predict from Eq. (10), since for these results the temperature was determined from a least-squares fit and not from the slope of the characteristic at the plasma potential.

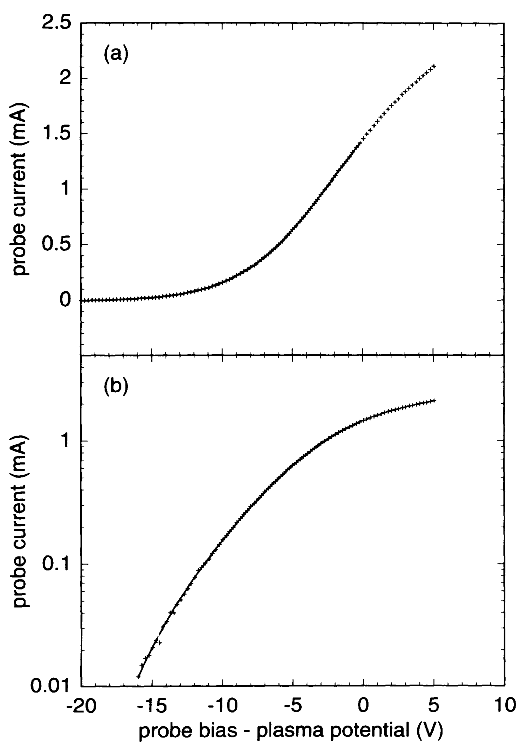

FIG. 6. Comparison between the current predicted from the drifting electron model (including an ion current term proportional to $\sqrt{V_r}$) and a Langmuir-probe characteristic taken in a sputtering magnetron discharge. Experimental data are indicated with crosses, and the fit by the solid line. In (a) the data and the fit curve are plotted on linear axes, while in (b) they are plotted on semilogarithmic axes. The agreement is excellent; the data and the fit fall on top of each other. Plasma parameters found are $V_p=-0.038$ V, $T=1.33$ eV, $n=8.22\times10^{15}$ m^{-3}, and $v_d=1.34\times10^6$ m/s, giving $v_d/v_{th}=1.96$.

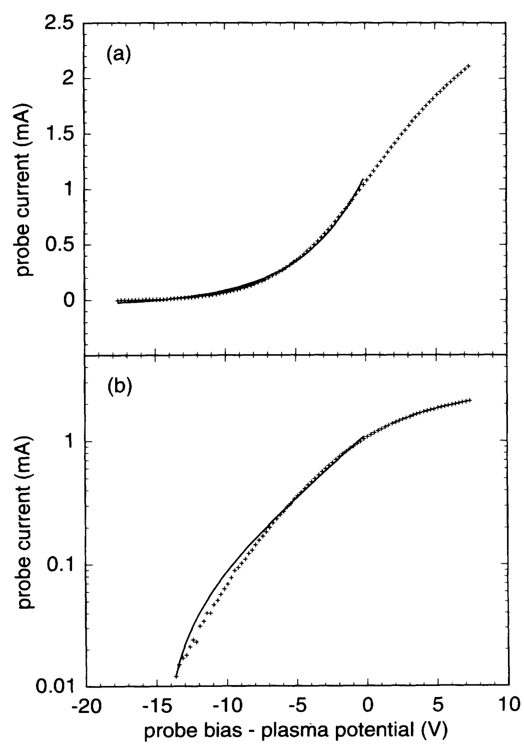

FIG. 7. The Langmuir-probe data from Fig. 6 fit by assuming that the electron distribution is stationary. As in Fig. 6, (a) is a plot on linear axes while (b) is on semilogarithmic axes. Experimental data are shown with crosses and the fit is given by the solid line. Plasma parameters from the stationary fit are $V_p=-2.35$ V, $n=8.30\times10^{15}$ m^{-3}, and $T=4.47$ eV. Agreement in this case is not nearly as good as that exhibited in Fig. 6, where a nonzero electron drift was assumed. Note in particular that the electron temperature is significantly overestimated.

V. SUMMARY

We have investigated a model describing the current collected by a cylindrical probe operating in the retarding field region for a low-pressure discharge in which the electrons have a net drift with respect to the probe. The degree of the drift is parametrized by the ratio of the electron drift to thermal velocities v_d/v_{th}. We find that for $v_d/v_{th} \lesssim 0.3$, the current collected from a drifting distribution is indistinguishable from that collected from a stationary distribution. Further, for $v_d/v_{th} > 1.25$, the inflection point of the probe characteristic underestimates the plasma potential. Finally, if $v_d/v_{th} \gtrsim 0.1$, fitting the characteristic with a stationary model significantly overestimates the electron temperature.

We have also shown how to use the drifting model to find the plasma parameters from experimental probe characteristics. Data from a sputtering magnetron discharge were fit using both the stationary and drifting electron models. The drifting electron model shows excellent agreement with the data, whereas the agreement between the stationary model and the data is poor.

ACKNOWLEDGMENT

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